Chapter-1

Diffential Equations

An equation involving derivatives of one or more dependent variables with respect to one or more independent variables is called a differential equation.

For examples: (1)

are differential equations.

Ordinary differential equations

A diffential equation involving ordinary derivatives (with only one independent variable)

is called ordinary differential equation.

Partial differential equations

A diffential equation involving partial derivatives (with more than one independent variable)

is called partial differential equation.

Equations (1) and (2) are ordinary diffential equations and equation (3) is a partial diffential equation.

Order and Degree of diffential equations

The order of the highest order derivative in the diffential equation is called the order of the diffential equation.

The power of the highest order derivative in the diffential equation is called the degree of the diffential equation.

For examples: (1)

1st equation is of order 1 and degree 2 Whereas the 2nd equation is of order 2 and degree 1.

Mathematical Model

Mathematical modeling which involves the following:

1. The formulation of a real-world problem in mathematical terms;

that is, the construction of a mathematical model.

2. The analysis or solution of the resulting mathematical problem.

3. The interpretation of the mathematical results in the context of the original real-world situation

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Real world situation | | | | | | |
|  | Formulation | |  | Interpolation | |
| Mathematical  Model | Mathematical  analysis | | | Mathematical  results |

Example of mathematical model

Newton’s law of cooling: The time rate of change of the temperature T (t) of a body is proportional to the difference between T and the temperature A of the surrounding medium.

i.e. ,

where k is a positive constant. Observe that if T > A, then dT/dt < 0, so the temperature is a decreasing function of t and the body is cooling. But if T , dT/dt so that T is increasing.

Thus the physical law is translated into a differential equation. If we are given the values of k and A, we should be able to find an explicit formula for T(t) and then with the aid of this formula—we can predict the future temperature of the body.

Nature of solution of differential equations

If we put y = mx in the diffential equation it is satisfied by it for every value of m.

So y = mx is called general solution. Similarly y = mx + c is the general solution of .

Thus a general solution contain same number of arbitrary constants as the order of the diffential equation. If we take m = 2 and c = 5 then the solution is called a particular solution, which is obtained by putting particular value of arbitrary constants.

A solution of the form y = some function of x is called explicit solution.

whereas a solution of the form x2+y2 = a2 is called implicit solution.

Form example,

y = 2sinx+3cosx is an explicit solution of the diffential equation

. and

is an implicit solution of .

Some examples

1. Show that is a solution of

Solution: We have

Putting these values in L.H.S. of the given equation,we have

) = = 0 =R.H.S.

is a solution of .

1. Find the value of r such that y = is the solution of .

Solution: We have r ,

Putting these values, we get,

r r= 0,

r = 0

.

Thus we see that are two solutions of

1. Show that is a solution of and hence find a value of C

so that .

Solution: Given

= 0

is a solution of .

Again using initial condition , we have,

is a particular solution of

Formation of differential equations.

Note : Differentiate the given equation as many times as the number of arbitrary

constants are involved in the equation and then eliminate the constants from

the equations so obtained and the given equation.

Example-1. The slope of the tantent to the graph of a function at a point (x, y) is the sum of x

and y, Find the differential equation.

Solution: We know that is the slope of the tangent at (x, y) of the graph.

According to question,

Which is the required differential equation.

Example-2. The line tangent to the graph of a function at the point (x, y) intersects the x-axis

at the point (x/2, 0), Find the differential equation.

Solution: We know that any line intersecting x-axis at (x/2, 0) is

Example-3. The time rate of change of population P is proportional to the square root of P.

Find a differential equation as a mathematical model.

Solution: The time rate of change of population P is .

According to question,

Example-4. Form a differential equation by eliminating the arbitrary constant m from the

equation

Solution: Given -------- (1)

Differentiating (1) with respect to x, we get,

.

Example-5. Form a differential equation by eliminating the arbitrary constants m and c

from the equation .

Solution: Given -------- (1)

Differentiating (1) with respect to x, we get,

Example-6. Find a differential equation of which is a solution.

Solution: Given equation is -------------- (1)

Differentiating with respect to x, we get,

--------------------- (2)

Again differentiating with respect to x, we get,

--------------- (3)

. ( Diving both sides by b)

Exercise: Find a differential equation of which

1. is a solution.
2. is a solution.

is the solution of

and find a value of c such that

Chapter-2

Solutions of first order and first degree Differential Equations

A first order and first degree differential equation can be written in the form

----------- (1)

If R.H.S. of (1) is a function of x alone, then it takes the form

---------- (2)

Which can be solved by directly integrating.

Therefore, solution of (2) is given by

y(x) =

where C is constant of integration.

i.e. y(x) = G(x)+C ---------- (3)

To satisfy an initial condition y(x0) = y0 , we put x = x0,  and y = y0. in (3),we get,

C = y0 - G(x0),

Putting the value of C in we get a particular solution. Which will satisfy the initial value problem,

; y(x0) = y0.

Example:- Solve the initial value problem

Solution:- Given

Integrating we get +C

----------- (1)

Putting x = 0 and y = 3 in (1) , we get (Using initial condition )

Exercise:- Solve the following initial value problems;



Exact differential equations

Any first order and first degree differential equation can be written in the form

.

Or . --------- (1)

The differential equation (1) is called exact differential equation if the expression

is exact differential in a domain D such that it is equal to

the total differential d F(x,y) for some function F in D.

that is the expression is an exact differential in D

if there exists a function F such that

for all (x,y) D.

For example is exact differential equation because if

we take

, then we see that

and .

i.e. . --------- (2)

So integrating (2),

we get,

Theorem: The necessary and sufficient condition for the differential equation

to be exact is that

where M and N have continuous first partial derivatives at all points in a domain D.

Proof :

--------- (1)

Then by definition there exists a function F such that

for all (x,y) D

and

Using the continuity of the first derivatives of M and N , we get

for all (x,y) D

*Sufficient condition*

Let us now suppose that

for all (x,y) D

To prove that the given equation is exact. i. e. we need to prove that there exists a function F such that

for all (x,y) D

Let

Then = as

=

Integrating with respect to x we get

N =

Now using the value of M and N we have

M dx + N dy

where d g(y) = f(y) dy. (say)

.

Hence M dx + N dy is exact.

Note : Now we depict a method to solve exact differential equations.

Example: Solve .

Solution : Given . …… (1)

Comparing (1) with M dx + N dy + 0 , we have

Thus

So we need to find a function F(x,y) such that

From these we have

F(x,y) =

From this we get

But = N =

, where c1 is arbitrary constant.

Hence

F(x,y) =

But one parameter solution is F(x,y) =

where C =

Which is the required solution of the given equation.

Example: Solve

Solution: Comparing the given equation with

We have and

Thus

.

Integrating M with respect to x keeping y as constant, we get,

The term free from x in N is , so integrating it with respect to y, we get ,

Hence the required solution is

, where c is an arbitrary constant.

Note: The above problem is solved by alternative method